Surrogate-Assisted Optimization of a 1D Heterogeneous Bar

MECH 293 Final Project

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Motivation

Why this project?

- Structural optimization is expensive
- Each design evaluation needs FEA
- Surrogate models (NN, GP, etc.) can approximate FEA

Goal

- Use a simple 1D bar problem
- Build a neural-network surrogate for compliance and mass
- Compare direct FEA search vs surrogate-assisted search

Problem Setup

1D Heterogeneous Bar

- Length L = 1, area A = 1, load F = 1
- Fixed at left end, traction at right end
- 10 elements, element-wise Young's modulus Ei

Optimization problem

- Minimize compliance C = Fu(L)
- Subject to M(z) ≤ 0.7 M (fully stiff)

Design variables

- Ei ∈ [1,10] for each element
- Density–modulus tradeoff:

$$\rho_i = 1 + \left(\frac{E_i - 1}{10 - 1}\right)^2 \cdot 2$$

- High Ei becomes disproportionately heavy
- Mass limit: $M \leq 0.7\, M_{full} = 2.1$

FEA Model

Linear 1D bar elements

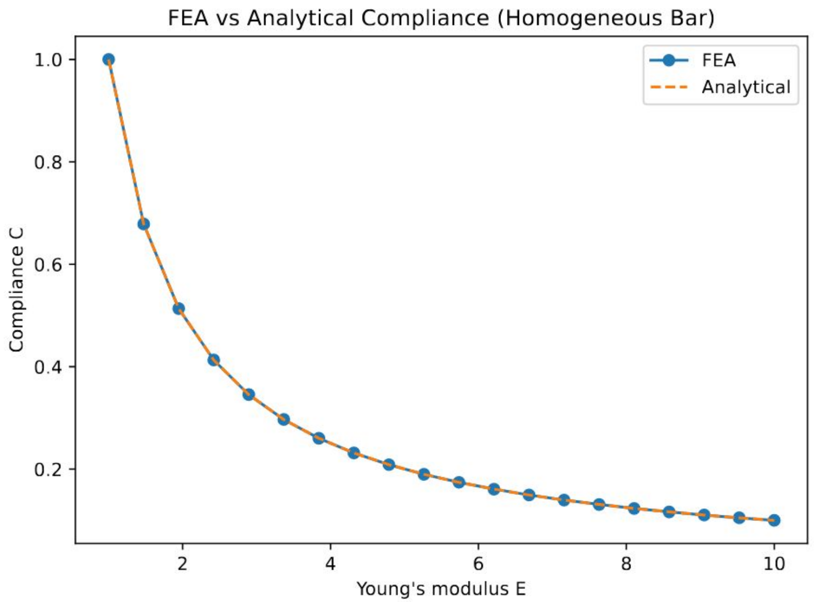
Local stiffness: $k^{(i)} = \frac{AE_i}{\Delta L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Assemble global K, apply u(0) = 0, solve for u

Verification:

Compare FEA with analytical solution:

$u(L) = FL/(AE)$, $C = F^2L/(AE)$, Max relative error ≈ 2×10^{-15}



Neural Network Surrogate

Dataset:

Random sample Ei ~ U(1,10)

Compute true (C,M) via FEA and density model

Train: 4,000 samples

Test: 1,000 samples

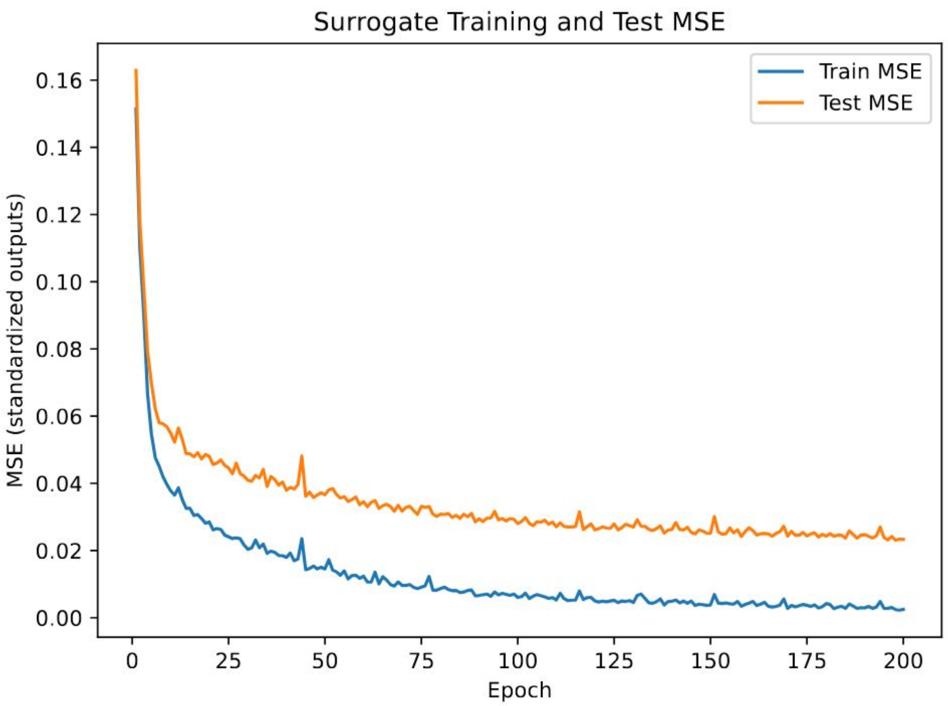
Network:

Fully connected MLP with 10 element input

Hidden: 3 layers × 64 ReLU

Output:

2 (Compliance, mass)



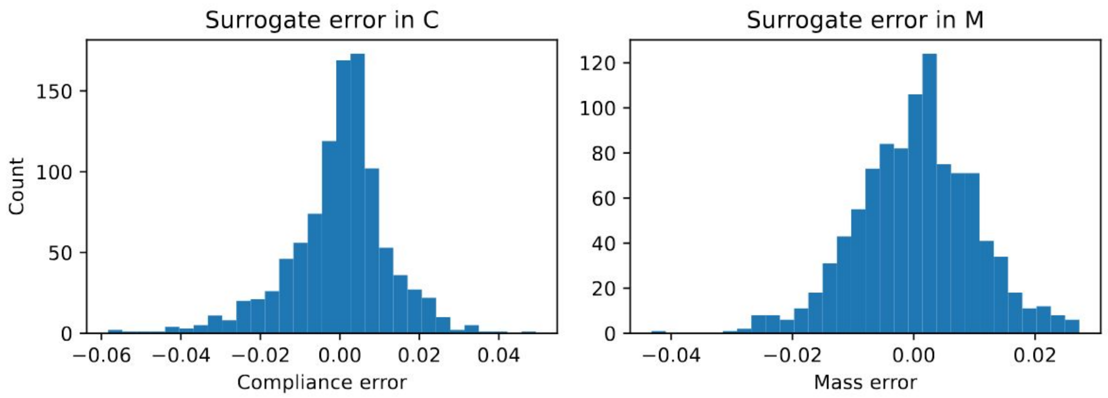
Surrogate Accuracy

Test set results:

RMSE in compliance ≈ 1.27×10^{-2}

RMSE in mass ≈ 9.77×10^{-3}

Errors roughly zero mean, light tails



Good regression accuracy overall,
but small errors in C can still change design ranking.

Optimization Strategies

Direct FEA random search

2,000 random designs

evaluate each with FEA

Keep designs with M ≤ Mmax

Choose lowest compliance design

Surrogate method

20,000 random designs

Evaluate with surrogate

Filter by predicted mass M^ ≤ Mmax

Take top 50 by predicted C^

Re-evaluate only these 50 with FEA

Baseline Results

Method	C_{best}	M_{best}	FEA calls	Surrogate calls
Direct random search	0.140773	2.086059	2000	0
Surrogate pre-screening	0.135981	2.054015	50	20000

- Surrogate-assisted method: better compliance, similar mass
- FEA calls reduced by ~40×

Discussion

- Surrogates can meaningfully reduce FEA calls.
- Two-stage scheme (screen + re-eval) is safer than surrogate-only.

Limitations:

- Very small 1D problem, FEA is cheap here.
- Only random search, no gradient-based or Bayesian optimization.

Thank you! Questions?